UNDERSTANDING MEASUREMENT UNCERTAINTY

by Peter Baker

PREMISE

One of the most important (and the most misapplied) aspects of industrial instrumentation technology is the terminology used in the specifications found on product data sheets. Terms like, linearity, repeatability, total probable error, traceability and the "accuracy statement" or stated another way "the inaccuracy statement" are often improperly used by both users, manufacturers and salespeople. The last term, the accuracy statement, is perhaps the most confused of all the terms. The confusion surrounding the accuracy specification, less commonly referred to as "total probable error", is compounded by the different ways manufacturers tend to write their specifications. Statements like, "percent of rate/reading (magmeter), percent of span (pressure transmitter) or a combination of the rate and span including an absolute error expressed in +/- velocity units or +/- 1 digit when the measurement is less than a percentage of the upper range limit (vortex meter)" are all "accuracy statement" specs that can be found on product specification sheets. What is required to assist in interpreting the data resultant from a measurement system is an understanding of and an agreement on, the terminology used. The intent of this paper is to present the method accepted as an international standard.

INTRODUCTION

The purpose of a measurement system is to numerically characterize the state or the performance of a physical process. Based on this measurement, one can then perform further action. To perform these measurements we use instrumentation and subsequently evaluate the performance of the instrument based on specifications printed in the manufacturer's product data sheets. However, as mentioned earlier, due to the confusing array of terms used to express a measuring instrument's performance, a standard method of evaluating the measurement itself is required.

Measurement uncertainty analysis is a method of numerically characterizing the state of performance of an instrument's measurement or that of an instrument system. Measurement uncertainty analysis, like I.S.O. 9000 certification, can be likened to a journey. The first step of that journey is a realization that there is an uncertainty in every measurement ever performed. This error is passed on to the instrument. Likewise the standards against which this aforementioned standard were calibrated contain an element of measurement uncertainty and so on back up the chain. Measurement uncertainty analysis permits the measurer to quantify and qualify the error and therefore predict a known end result upon which process decisions can be based. Once one accepts and understands that there is indeed error present in any measurement we make, the next step is to classify the error.

ERROR CLASSIFICATIONS

There are 2 types of error. There is precision or random error and systematic or bias error.

PRECISION or RANDOM ERROR

Precision error is characterized by the random scattering of the data about the true measurement value. The precision index component of a measurement uncertainty analysis is statistically derived from the Gaussian or normal distribution of error data. (Note: This is why pressure transmitters are spec'd as % of span). The higher quality the instrument, the tighter the installation parameters and the higher up the calibration chain the instrument is to the primary standard the smaller the random error should be. However, by simple definition once the calibration of an instrument is completed, the precision error becomes fixed and is transmitted to elements lower in the chain as systematic error. The random error component of a measurement uncertainty analysis is calculated, using the root-sum-square method, as the standard deviation of the measurement results. The result is then multiplied by the appropriate students "t" value to ascertain the confidence limits for a particular instrument. The students "t" value is determined by the degrees of freedom. The degrees of freedom are a function of the number of samples of data and the precision indices of the precision error sources (when a number of error sources are involved the Welch-Satterthwaite formula is used to combine the degrees of freedom). Generally, where the degrees of freedom are greater than 30, a students "t" value of 2.0 can be used for a confidence level of 95%. I.S.O. Standard 5168 documents the appropriate "t" that should be used for different sample sizes.

SYSTEMATIC or BIAS ERROR

Systematic error is more difficult to determine and is usually estimated based on experience and observation. There are 5 types of bias error and are they are classified as follows based on the error's sign (+/-) and magnitude:

1/ known sign, known large (maybe) magnitude

-removed as a factor during calibration

2/ known sign, known small magnitude

-this factor can usually be ignored however should be documented i.e. measurement uncertainty of primary standard causing bias

3/ unknown sign, unknown large magnitude

-it is expected that installing the instrument as per accepted and proper engineering practices would remove this uncertainty

4&5/ (grouped together) known & unknown sign, unknown small magnitude

-these are the most difficult to ascertain and can include non-symmetrical bias. Some methods of estimating these errors include testing the instrument at different calibration facilities, transporting a calibration standard to the site (i.e. refinery in-situ ball provers) and using independent alternate methods of measuring the same parameter. (i.e. a flow may be measured by a flowmeter, draw-down or pump speed)

Once the systematic error data has been collected it is combined and reduced to a single number using the root-sum square method common in statistics. Non-symmetrical bias limits warrant special consideration and are beyond the scope of this paper.

To summarize, the decision to classify the error observed as precision or bias should be made based on the results recorded and nothing else. If a scatter of the results is seen then the error is classified as random. If the results show an offset then the error is systematic. Once the error that may be present and the source of it is understood, a measurement uncertainty analysis can begin.

THE MEASUREMENT UNCERTAINTY ANALYSIS

Measurement uncertainty analysis, like trouble-shooting involves gathering as much information about the measurement system as is practicable. After categorizing the error, the errors may be grouped as per their source. The 4 common sources of error are:

1/ calibration errors (include the bias error due to the calibration chain here)

2/ data acquisition errors (interpretation or reading errors etc.)

3/ data reduction errors (including rounding off, truncation & approximations)

4/ methodology errors (sampling errors, instrument location error etc.) Organizing the data in tabular form helps ensure that nothing is missed when reducing the information. The collection and organization of the data used to calculate the precision error and the systematic error components can be done by two methods. The first method is to acquire and calculate the data separately by both its category (precision & bias) and its grouping (2 sets of data 4 times, separated as per error source). This first method is both time consuming and expensive. The second method is to collect the data separately by category but group all the error sources together. The second method is very acceptable and also economically advantageous! Once all the data is collected and tabulated, the errors (random & bias) are each calculated by the root-sum-square method. The only calculation that remains is to combine the random error component and the systematic error component to compute the final measurement uncertainty statement or uncertainty limit. There are two methods of doing this. The U_{ADD} method (adding the random and the bias error together) and the U_{RSS} method (root-sum-square of the random and the bias error) as shown in the equations below. The U_{ADD} method gives a 99% confidence level and the U_{RSS} method results in a 95% confidence level. I.S.O. and A.S.M.E have adopted the second method. The U_{RSS} method assumes 30 degrees of freedom and as a result produces a 95% confidence level. If the degrees of freedom are less than 30 reference to a standard table of Student's "t' value can be used to determine the confidence results. The mathematical representations of these methods are shown below:

$$U_{RSS} = +/- [B + t_{95}S]$$

$$U_{ADD} = +/- [B^2 + (t_{95}S)^2]^{1/2}$$

Where:

B = the bias limit of the results

S = the precision index of the results

t_{95} = the Student's t for degrees of freedom

$t_{95}S$ = the random error component of the uncertainty

ERROR PROPAGATION

So far, the methodology discussed assumes that only one variable is involved. For example a group of pressure measurements or an array of thermocouples. However, often the instrument measuring system seldom functions independently. Consider an example of a steam mass flow measuring system. Such a system would involve a square law density dependent flow element, a pressure transmitter (let's leave the square root extractor out of it for now as it introduces additional complications!), a temperature transmitter and a gauge pressure transmitter. The measurement uncertainty analysis would therefore involve a variety of units (delta P, ^{0}C, PSIA etc.). In order to correctly combine the results data when mixed units are involved, error propagation must be used. There are 3 methods of error propagation. They are:

1/ Taylor's Series

2/ "dithering"

3/ Monte Carlo simulation

Method 2 &3 involve extensive, complex and detailed calculations that are usually worked out on a computer. The Taylor Series method, while still relatively complex, can be performed by a person with reasonable mathematical ability. The Taylor Series method entails developing sensitivity or influence coefficients and applying them to both the precision and the systematic error components of each instrument prior to combining the data.

THE REPORT

One can perform the most detailed technical analysis in the world only to have it "fall flat" due to a poor presentation format. In today's high paced world it is not only contents but looks and format that count. The KISS principle, "Keep It Simple Simpleton" is a good rule to follow. A simple presentation of the measurement uncertainty statement followed by the final data used in the analysis in table form is a good format to follow. The presentation of one's results may also reflect and be influenced by legal standards (i.e. Consumer & Corporate Affairs' standards for Custody Transfer). The bottom line though, is to be consistent, clear and precise, much as you hope the measurements are.

CONCLUSION

The tighter operating tolerances and controls strategies of today's processing plants and the higher standards of environmental emissions monitoring require increasingly complex and sophisticated instrumentation systems. In order to fully understand and apply these systems, it is necessary to have a method of clearly and precisely communicating the systems performance characteristics. Measurement uncertainty analysis, when properly applied, is a non-ambiguous method of achieving this goal.

REFERENCES:

Furness, R.A. Fluid Flow Measurement, Longman Group UK Ltd., 1989

Dieck, R. H. Measurement Uncertainty, Instrument Society of America, 1992

Mendenhall, W. Statistics for Engineering & the Sciences, Dellen Publishing 1992

ANSI/ASME PTC 19.1 1985 Performance Test Code, Instruments & Apparatus, Part I, Measurement Uncertainty